PROBLEMS

11-3 Simple Harmonic Wave

1. A wave has an angular frequency of 110 rad/s and a wavelength of 1.8 m. Calculate (a) the angular wave number and (b) the speed of the wave.

2. A sinusoidal wave of frequency 500 Hz has a speed of 350 m/s. (a) How far apart are two points that differ in phase by $\pi/3$ rad? (b) What is the phase difference between two displacements at a certain point at times 1.00 ms apart?

3. A note of 440 Hz is played on a violin submerged in a swimming pool at the wedding of two scuba divers. Given that the speed of compression waves in pure water is 1498 m/s, what is the wavelength of that tone?

4. (a) Write an equation describing a sinusoidal transverse wave traveling on a cord in the $+x$ direction with a wavelength of 10 cm, a frequency of 400 Hz, an amplitude of 2.0 cm. (b) What is the maximum speed of a point on the cord? (c) What is the speed of the wave?

5. A transverse sinusoidal wave of wavelength 20 cm is moving along a string in the positive $x$ direction. The transverse displacement of the string particle at $x = 0$ as a function of time is shown in Fig. 11-22. (a) Make a rough sketch of one wavelength of the wave (the portion between $x = 0$ and $x = 20$ cm) at time $t = 0$. (b) What is the speed of the wave? (c) Write the equation for the wave with all constant evaluated. (d) What is the transverse velocity of the particle at $x = 0$ at $t = 5.0$ s?

6. The profile of a transverse harmonic wave on a long taut nylon thread is described in SI units by function $y = 0.04\sin 2\pi x$. Given that the wave travels at a speed of 2.0 m/s, determine the maximum transverse acceleration of any point on the thread.

7. Show, for a sinusoidal transverse wave traveling on a string, that the absolute value of the string’s slope at any point $x$ is equal to the ratio of the transverse speed of the particle to the speed of the wave at that point.

8. Determine if the function $y = y_m \sin kx \cos \omega t$ is a solution of the wave equation.

9. Show by direct substitution that the following functions satisfy the wave equation:
   (a) $y(x, t) = y_m \ln(x + vt)$; (b) $y(x, t) = (x - vt)^4$.

11-4 Speed of a Traveling Wave

10. What is the speed of a transverse wave in a rope of length 2.00 m and mass 60.0 g under a tension of 500 N?

11. The linear density of a string is $1.6 \times 10^{-4}$ kg/m. A transverse wave on the string is described in SI units by the equation $y = 0.021\sin(2.0x + 30t)$. What are (a) the wave speed and (b) the tension in the string?

12. A sinusoidal transverse wave of amplitude $y_m$ and wavelength $\lambda$ travels on a stretched cord.
(a) Find the ratio of the maximum particle speed (the speed with which a single particle in the cord moves transverse to the wave) to the wave speed. (b) If a wave having a certain wavelength and amplitude is sent along a cord, would this speed ratio depend on the material of which the cord is made, such as wire or nylon?

13. A string has a mass per unit length of 2.5 g/m and is put under a tension of 25.0 N as it is stretched taut along the y-axis. The free end is attached to tuning fork that vibrates at 50.0 Hz, setting up a transverse wave on the string having amplitude of 5.00 mm. Determine the speed, angular frequency, period, and wavelength of the disturbance.

14. A heavy nylon guitar string with a linear mass density of 7.5 g/m is stretched to a tension of 80 N, what is the speed of the transverse wave that can be generated on the string? What tension will be required to double the speed?

15. A uniform rope of length \( L \) and mass \( m \) hangs freely straight down from a hook in the ceiling. (a) Show that the speed of a transverse wave on the rope is a function of time \( y \), the distance from the lower end, and is given by \( v = \sqrt{gy} \). (b) Show that the time a transverse wave takes to travel the length of the rope is given by \( t = 2\sqrt{L/g} \). (c) What is the maximum speed of the wave if a mass \( M \) is hung on the bottom of the rope?

11-5 Energy Transported by Waves

16. Calculate the kinetic energy and potential energy contained in one wavelength of a harmonic traveling wave on a string whose mass density is \( \mu \) and whose tension is \( F_T \).

17. A sound wave has an intensity of \( 1.7 \times 10^{-6} \) W/m². Given that the speed of sound in air is 330 m/s, what is the energy density of the sound wave?

18. The intensity of the sunlight that enters earth’s atmosphere is 1.38 kW/m². If about 60 percent of this sunlight reaches the earth’s surface, what is the total energy that reaches the earth’s surface in one year? How many 400 MW power plants would it take to supply the same amount of energy? [Hint: treat the sun as a distant point source. The intensity is the power/m² perpendicular to the direction in which the sunlight travels.]

11-6 Superposition of Waves and Standing Waves

19. Show that the superposition of the two traveling waves \( \sin(kx - \omega t) \) and \( \cos(kx - \omega t) \) can be written in the form \( A \sin(kx - \omega t + \phi) \), and find \( A \) and \( \phi \).

20. Two identical traveling waves, moving in the same direction, are out of phase by \( \pi/2 \) rad. What is the amplitude of the resultant wave in terms of the common amplitude \( y_m \) of the two combining waves?

21. A nylon guitar string has a linear density of 7.2 g/m and is under a tension of 150 N. The fixed supports are 90 cm apart, shown in Fig. 11-23. Calculate the (a) speed, (b) wavelength, (c) frequency of the traveling waves whose superposition gives this standing wave.

Fig. 11-23 Problem 21.

22. Show theoretically that for a string fixed at both ends, its standing-wave frequencies are
given by
\[ f_n = \frac{1}{2} n \sqrt{\frac{F_T}{Lm}}, \]
where \( L \) is its length and \( m \) its mass.

23. A guitar in tuned by adjusting the tension in its strings. Show that with a change in tension there is a corresponding frequency change given by \( \frac{df}{f} = \frac{1}{2} \frac{dF_T}{F_T} \).

24. If a violin string vibrates at 440 Hz as its fundamental frequency, what are the frequencies of the first four harmonics?

25. If two successive harmonics of a vibrating string are 280 Hz and 350 Hz, what is the frequency of the fundamental?

26. The displacement of a transverse wave traveling on a string is represented by
\[ y = 4.2 \sin(0.71x - 47t + 2.1), \]
where \( y \) and \( x \) are in cm and \( t \) in s. (a) Find an equation that represents a wave which, when traveling in the opposite direction, will produce a standing wave when added to this one. (b) What is the equation describing the standing wave?

11-7 Sound Waves

27. For a spherical traveling wave uniformly away from a point source, show that the displacement can be represented by
\[ y = \frac{y_m}{r} \sin(kr - \omega t), \]
where \( r \) is the radial distance from the source and \( y_m \) is a constant.

28. A faint sound with an intensity of \( 10^{-9} \) W/m² is measured by a sound-level meter. What will the reading be in dB?

29. Someone playing a CD at 60 dB wants to make the music twice as loud. At what sound level should it be played?

30. The intensity of a sound is tripled. By how many decibels does it increase?

31. What is the sound level 10 m away from a point source radiating 1.2 W of acoustic power?

11-8 Beats

32. The sound from a tuning fork of 1000 Hz is beat against the unknown emission from a vibrating wire. If beats are heard at a frequency of 4 Hz, what can be said about the frequency of the wire?

33. On tapping two tuning forks, an observer hears a succession of intensity maximum arriving at a rate of one every 0.99 s. What is the difference in frequency between the two forks?

11-9 The Doppler Effect

34. A police car, its siren blaring at 1000 Hz, is traveling at 20.00 m/s while chasing a garbage truck moving at 15.00 m/s a block in front of it. What the apparent frequency will the garbage collectors hear? The speed of sound is 330.0 m/s.

35. A man running toward the stage in a theater hears an A₄ note from a stationary tuning fork to have a frequency of 441 Hz instead of its normal 440 Hz. About how fast is he going?

36. An observer with a stationary source of sound sends out a signal directly toward an approaching target. Write an expression for the frequency of the beats heard by the
Problems
1. (a) 3.49 rad/s; (b) 31.5 m/s.
2. (a) 11.7 cm; (b) π rad.
3. 3.40 m.
4. (a) \(y(x, t) = 2.0\sin2\pi(0.10x - 400t)\), with \(x\) and \(y\) in cm and \(t\) in s; (b) 50 m/s; (c) 40 m/s.
5. (a)
   \[
   y(x, t) = 2.0\sin2\pi(0.10x - 400t), \text{ with } x \text{ and } y \text{ in cm and } t \text{ in s;}
   \]
   (b) 2 cm/s; (c) \(y = 4.0\sin(0.1x - 0.2t)\), with \(y\) and \(x\) in cm and \(t\) in s; (d) 2.5 cm/s.
6. 6.3 m/s².
8. The function is a solution.
10. 129 m/s.
11. (a) 15 m/s; (b) 0.03 N.
12. \(2\pi\omega/\lambda\); (b) no.
13. 100 m/s, 314 rad/s, 0.020 s, 2.00 m.
14. \(v = 1.0\times10^2 \text{ m/s}; F_T = 3.2\times10^2 \text{ N when } v\) is doubled.
15. \(\sqrt{\frac{gL + MgL / m}{\pi^2}}\); (c) \(v_{max} = \sqrt{gL + MgL / m}\) .
16. \(K = U = \pi^2 y_m^2 F_T / \lambda\).
17. 5.2×10⁻⁹ J/m³.
18. 3.32×10¹⁴ J, 2.65×10⁸ power plants.
19. \(\sqrt{2}, \pi / 4\).
20. \(1.4y_m\).
21. (a) 144 m/s; (b) 60 cm; (c) 240 Hz.
24. 440 Hz, 880 Hz, 1320 Hz, 1760 Hz.
25. 70 Hz.
26. (a) \(y_2 = 4.2\sin(0.71x + 47t + 2.1)\);
    (b) \(y = 8.4\sin(0.71x + 2.1)\cos(47t)\), with \(y\) and \(x\) in cm and \(t\) in s.
27. Proof problem.
28. 30 dB.
29. 63 dB.
30. 4.8 dB.
31. 90 dB.
32. 996 Hz or 1004 Hz.
33. 1.0 Hz.
34. 1016 Hz.
35. 0.75 m/s.

36. \( f_o - f_s = \frac{2v_t}{v-v_t} f_s \).